INTEGRATION OF A SPECTRAL BAROTROPIC MODEL FROM GLOBAL 500-MB. CHARTS

ANDRÉ J. ROBERT

Meteorological Service of Canada, Montreal, Quebec

ABSTRACT

This project used a series of 500-mb. charts prepared originally for the study of planetary waves. These charts, covering both hemispheres, provided the initial conditions for a spectral barotropic model. In this model, the calculations proceeded from functions equivalent to spherical harmonics with the stream field represented by 153 degrees of freedom. A set of five integrations carried to 72 hr. produced reasonably good forecasts that did not appear to be affected seriously by the deficiencies of the observational network.

1. INTRODUCTION

There is no doubt that general circulation studies will play an important role in the field of meteorological research over the next decade. The atmospheric models required for this particular type of research will have to be rather elaborate and complete in order to produce highly realistic time series.

Because of their inherent deficiencies, the present hemispheric numerical models with an artificial equatorial boundary, will soon give way to models spanning the entire earth. In fact, only a completely global model can adequately depict the behavior of the planetary waves and these waves have a considerable influence on other features of the general circulation.

The grid point method commonly used for the integration of the meteorological equations does not apply easily to the globe. In contrast, the more elaborate spectral method does not apply easily to only a portion of the earth's surface. In spite of a large number of additional difficulties, it appears that sufficient progress has been made with the spectral method in recent years to warrant its application to the present investigation.

For his extended numerical integrations, Baer [1] used a grid point model and some time later (Baer [2]) he performed a similar experiment with a specification of the variables in terms of spherical harmonics. He started his integrations from simple hypothetical initial conditions and both models required and retained symmetry with respect to the Equator at all times. Spectral forecasts were later prepared by Ellsaesser [6] from real data for the Northern Hemisphere. Only the odd spherical harmonics entered into the calculations and it is quite obvious that both even and odd functions must be considered for an adequate representation of the planetary modes.

Fully global spectral integrations were undertaken by the author (Robert [8]) as a logical extension of the work of Baer and Ellsaesser. For simplicity, this first calculation used hypothetical initial conditions but now the possibility exists of applying the same method to real data. The main objective of the present study has been to carry out such an experiment with the hope that it would reveal some of the characteristics of planetary waves that do not show up in hemispheric integrations.

Members of the Department of Meteorology at McGill University, under the direction of Prof. B. W. Boville, produced the charts used for the present series of tests. Steinberg [9] transformed the 30 global charts of the 500-mb. geopotential for September 1957 into spherical harmonic representations and Merilees [7] used the linear balance equation to convert the spectral versions of the geopotential into equivalent stream functions. Merilees generalized the method described previously by Eliasen and Machenhauer [5] and the resulting stream functions were used for this study.

The main purpose of this paper is to present the results of a few global forecasts prepared from real data with the hope that this will generate an interest in the method and encourage its development.

2. THE MODELING EQUATIONS

The barotropic model developed by Bolin [3] and Cressman [4] contains an empirical term applicable to the stabilization of the ultra-long waves. The version proposed by Bolin and used here without any additional refinements should give acceptable global integrations.

$$\frac{\partial Q}{\partial t}+J(\psi,\, Q)=\kappa^2\,\frac{\partial\psi}{\partial t} \tag{1}$$

$$Q=2\Omega\,\sin\varphi+\nabla^2\psi \tag{2}$$

$$\kappa^2=\frac{2\mu\Omega^2}{gH} \tag{3}$$

where:

ψ is the stream function,
Q the absolute vorticity,
φ the latitude,
Ω the angular velocity of rotation of the earth,
g the acceleration of gravity, and
H the average height of the 500-mb. surface.

The symbols J and ∇^2 represent the Jacobian and Laplacian operators respectively. To the empirical constant μ we give the value $\mu=4$ as suggested by Cressman.

The input to the model consists of 500-mb. global analyses of the geopotential. The stream functions are generated from the linear balance equation

$$\nabla\cdot(2\Omega \sin \varphi \nabla\psi)=\nabla^2\phi. \tag{4}$$

Forecasts of the stream function are then prepared with equation (1) and finally the linear balance equation is used in reverse to produce a forecast of the geopotential.

The integration of the model described above presents no problems if we follow a scheme involving the functions

$$G_n^m(\lambda, \varphi)=e^{im\lambda}\cos^M \varphi \sin^n \varphi \tag{5}$$

where λ represents the longitude and M is simply the absolute value of m. We may then form truncated series of these functions that will describe the stream function

$$\psi=\sum_{m=-8}^{+8}\sum_{n=0}^{8} A_n^m G_n^m(\lambda, \varphi). \tag{6}$$

This series contains 153 terms usually presented in the form of a table. Only the amplitudes A_n^m need to be tabulated and the various meteorological calculations simply require a manipulation of these tables of numbers. This method, previously described by the author (Robert [8]), applies quite well to the integration of numerical models of the atmosphere. In the present study, we will give no further consideration to the method itself and concentrate our attention on the numerical experiments.

The McGill group analyzed the 30 global charts of the stream function in terms of spherical harmonics generated from normalized associated Legendre polynomials of the first kind:

$$\psi=\sum_{m=-8}^{+8}\sum_{n=0}^{+8} C_n^m e^{im\lambda}P_{M+n}^M \tag{7}$$

but the spherical harmonics may be converted to the functions selected in (5)

$$e^{im\lambda}P_{M+n}^M=\alpha_n^M\sum_{j=0}^{\infty}\frac{(-1)^j(M+n-j-\frac{1}{2})!}{4^j j!(n-2j)!}G_{n-2j}^m. \tag{8}$$

This series contains only a finite number of terms since all terms where $j>n/2$ vanish because of the last factorial in the denominator. In (8)

$$\alpha_n^M=\frac{2^{M+n-1}}{(\frac{1}{2})!}\left[\frac{(M+n+\frac{1}{2})n!}{(2M+n)!}\right]^{1/2}. \tag{9}$$

With the help of equations (8) and (9), the conversion of the coefficients C_n^m to A_n^m gives no difficulties. The correspondence between spherical harmonics and the functions selected in (5) shows up clearly in equation (8). This correspondence extends also to the numerical integrations. Calculations performed in terms of either set of functions would yield identical results indicating that the two methods are equivalent.

3. RESULTS

The preparation of global forecasts on a routine basis would present some serious problems due to the poor data coverage over the equatorial belt and over the Southern Hemisphere. On the other hand, we do not require a very dense data network in order to depict and follow the very large-scale features of the atmosphere. It seems that if we omit the short waves and concentrate on the prediction of the behavior of the long waves, this will alleviate our data problem to the extent that it will no longer constitute a a major obstacle. Our inability to incorporate the effects of short waves on large-scale patterns should not have a serious effect on the forecasts.

The integrations performed with a truncation at wave number eight do not contain all the meteorological disturbances in the synoptic range of wavelengths but nevertheless the resulting predictions show some skill. A quick glance at a few charts will help us form an opinion about the merits and other characteristics of the method. The stream function derived from an analysis of the geopotential for 1200 GMT September 5, 1957, is presented in figure 1. The first 153 terms in the series (spherical harmonics) representing the stream function were used to generate this chart. Patterns in the stream function over the equatorial belt correspond to weak features in the geopotential and will not show as much continuity in time as one will observe at higher latitudes.

The 48-hr. barotropic forecast prepared from the chart of figure 1 is shown in figure 2. The shorter wave patterns embedded in the westerlies move eastward as in all barotropic models. The wave located near 45°S. and 40°W. in figure 1 moves to 20°W. during the integration period of 48 hr. On the other hand, all the systems located near the Equator show appreciable westward displacements. This is an interesting result since these motions take place in the absence of easterlies.

The perturbations located at the Equator are normally eliminated in hemispheric integrations or they are kept fixed by the boundary conditions. The results illustrated here can only be produced by a global integration or at least by a model that covers a broad range of latitudes centered at the Equator. The chart for 1200 GMT September 7, 1957, shown in figure 3 corresponds to the verification time of the forecast. There is no evidence on this chart of any substantial retrogression of the very long waves. Global integrations simply enhance this deficiency of the barotropic model.

The numbers appearing in table 1 are all about 10 to 15 m. lower than the figures that would come out if all scales of motion were considered. It must be pointed out here that the shorter waves have also been taken out of the analysis for the purpose of determining the differences

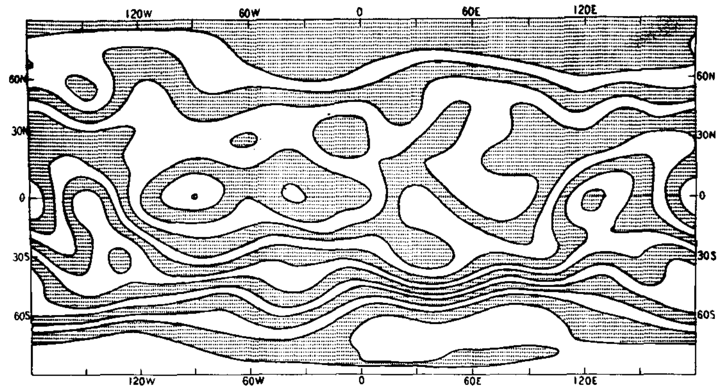

FIGURE 1.—The 500-mb. stream function in terms of longitude and latitude for 1200 GMT Sept. 5, 1957. The spacing between the contours is 12 km.² sec.⁻¹ The linear balance equation was used to produce the stream function from the geopotential.

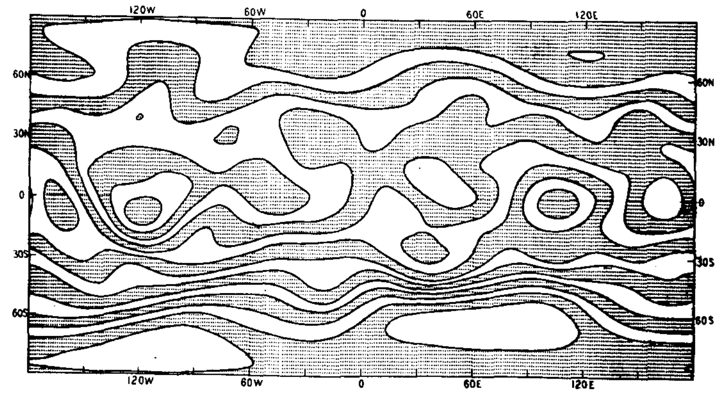

FIGURE 2.—The 48-hr. barotropic forecast of the stream function from 1200 GMT Sept. 5, 1957. The coordinates are longitude and latitude and the spacing between contours is 12 km.² sec.⁻¹

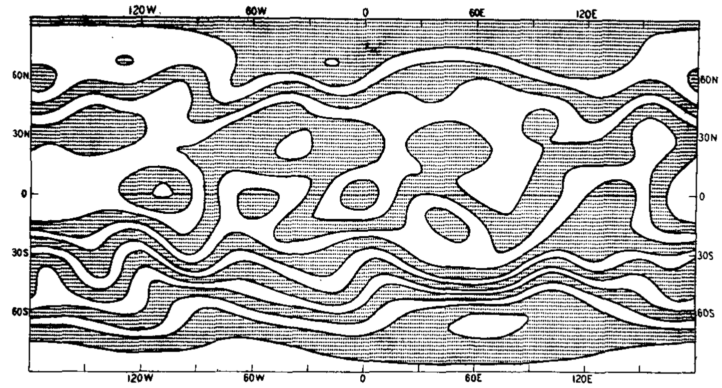

FIGURE 3.—The 500-mb. stream function in terms of longitude and latitude for 1200 GMT Sept. 7, 1957. The spacing between contours is 12 km. ² sec.⁻¹ The linear balance equation was used to produce the stream function from the geopotential.

between forecasts versus persistence scores. If the shorter waves were considered the numbers appearing in table 1 would increase and most likely the difference between forecast and persistence scores would also increase slightly. The global verification scores show an improvement of 5 m. over persistence forecasts. The operational barotropic models used over the Northern Hemisphere generally show an improvement of the order of 12 m. for the month of September.

4. CONCLUSION

The rapid progress made by the models based on spherical harmonics or any other equivalent spectral representation leaves little doubt about their future. The atmospheric simulators of the spectral type will soon start competing with the classical models that use the grid point method. A significant advantage of the spectral models resides in the facility with which they can produce global forecasts and the need for such predictions will rapidly develop during the coming years. The spectral method still offers many unexplored possibilities and continued research in this field will undoubtedly uncover even more promising applications.

TABLE 1.—*The root-mean-square error in the 48-hr. barotropic forecasts (F) of the 500-mb. height field against the 48-hr. root-mean-square height changes (P) at the 500-mb. level (persistence forecasts). The verification area covers both hemispheres completely and the scores are given in meters.*

Forecasts from	48 hr. (global)	
	F	P
Sept. 5, 1957	49.0	52.8
Sept. 10, 1957	49.3	53.6
Sept. 12, 1957	50.0	58.6
Sept. 17, 1957	41.9	46.7
Sept. 21, 1957	36.5	40.1
Mean	45.3	50.4

REFERENCES

1. F. Baer, "The Extended Numerical Integration of a Simple Barotropic Model," *Journal of Meteorology,* vol. 18, No. 3, June 1961, pp. 319–339.
2. F. Baer, "Integration With Spectral Vorticity Equation," *Journal of the Atmospheric Sciences,* vol. 21, No. 3, May 1964, pp. 260–276.
3. B. Bolin, "An Improved Barotropic Model and Some Aspects of Using the Balance Equation for Three-Dimensional Flow," *Tellus,* vol. 8, No. 1, Feb. 1956, pp. 61–75.
4. G. P. Cressman, "Barotropic Divergence and Very Long Atmospheric Waves," *Monthly Weather Review,* vol. 86, No. 8, Aug. 1958. pp. 293–297.
5. E. Eliasen and B. Machenhauer, "A Study of the Fluctuations of the Atmospheric Planetary Flow Patterns Represented by Spherical Harmonics," *Tellus,* vol. 17, No. 2, May 1965, pp. 220–238.
6. H. W. Ellsaesser, "Evaluation of Spectral Versus Grid Methods of Hemispheric Numerical Weather Prediction," *Journal of Applied Meteorology,* vol. 5, No. 3, June 1966, pp. 246–262.
7. P. Merilees, "Harmonic Representation Applied to Large Scale Atmospheric Waves," *Publication in Meteorology* No. 83, McGill University, Montreal, Sept. 1966, 174 pp.
8. A. J. Robert, "The Integration of a Low Order Spectral Form of the Primitive Meteorological Equations," *Journal of the Meteorological Society of Japan,* Tokyo, vol. 44, No. 5, Oct. 1966, pp. 237–245.
9. H. L. Steinberg, "A Spherical Harmonic Specification of the Global 500 Mb. Surface," M.S. Thesis, Department of Meteorology, McGill University, Montreal.

[*Received June 8, 1967, revised September 11, 1967*]